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SUPER EDGE ANTIMAGIC TOTAL LABELING ON DISJOINT UNION OF CYCLE WITH CHORD

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Abstract

A graph G with order p and size q is called (a, d)-edge antimagic total ((a, d)-EAT) if there exist integers $a < 0$, $d \geq 0$ and a bijection $\alpha : V \cup E \rightarrow \{1, 2, 3, \ldots, p + q\}$ such that $W = \{w(uv), uv \in E\} = \{a, a + d, \ldots, a + (q - 1)d\}$, where $w(uv) = \alpha(u) + \alpha(v) + \alpha(uv)$. An (a, d)-EAT labeling α of graph G is super if $\alpha(V) = \{1, 2, \ldots, p\}$. In this paper we describe how to construct a super (a, d)-EAT labeling on some classes of disjoint union from non isomorphic graphs, namely disjoint union of cycle with cycle with chord $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$.

Keywords : (a, d)-*edge*-antimagic *total labeling, super*(a, d)-*edge antimagic total la- belling, cycle with chord,* $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$.

1. Introduction

All graphs in this paper are finite, undirected, and simple. $V(G)$ and $E(G)$ (in short, V and E) stand for the vertex-set and edge-set of graph G, respectively. Let $e = \{u, v\}$ (in short, $e = uv$) denote an edge connecting vertices u and v in G. Then, let order $|V(G)|$ in G denoted by p and size $|E(G)|$ in G denoted by q.

A *labeling* of a graph is any mapping that sends some set of graph elements to a set of numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex-labelings* or *edge-labelings*. In this paper we deal with the case where the domain is $V \cup E$, and these are called *total labeling*. We define the *edge-weight* of an edge $uv \in E$ under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv. General references for graph-theoretic notions is [12]. A general survey of graph labelings is [6].

A graph G is called (a, d)-*edge antimagic total* ((a, d)−*EAT*) if there exist integers $a > 0$, $d \geq 0$ and a bijection function $f : V \cup E \rightarrow \{1, 2, \ldots, p + q\}$ such that the set of edge-weights is $w(uv) = f(u) + f(uv) + f(v)$, $uv \in E$, form an arithmetic progression $\{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$. In particular, an $(a, d) - EAT$ labeling of graph G is super if $f: V \rightarrow \{1, 2, \ldots, p\}$. Thus, a *super* (*a, d*)*-edge-antimagic total graph* is a graph that admits a (a, d)-edge-antimagic total labeling.

The concept of (a, d)-edge antimagic total labeling, introduced by Simanjuntak *et al*. in [14], is natural extension of the notion of *edge-magic* labeling defined by Kotzig and Rosa

[1] (see also [9], [13], [3] and [15]). The super (a,d)-edge-antimagic total labeling is natural extension of the notion of *super edge-magic* labeling which was defined by Enomoto *et al.* in [4]. In this paper we investigate the existence of super (a,d)-edge-antimagic total labelings for disjoint union of non isomorphic graphs. A number of classification studies on super $(a,d)-EAT$ (resp. $(a,d)-EAT$) for disjoint union of non isomorphic graphs has been extensively investigated. For instances, some constructions of super $(a,0)$-edge-antimagic total labelings for $nC_k \cup mP_k$ and $K_{1,m} \cup K_{1,n}$ have been shown by Ivančo and Lučkaničová in [7]. In [5] Sudarsana, Ismaimuza, Baskoro, and Assiyatun show that $P_n \cup P_{n+1}$, $nP_2 \cup P_n$, $(n \geq 2)$, and $nP_2 \cup P_{n+2}$ are super edge antimagic total labeling. Dafik *et al* also found disjoint union of non isomorphic graph which admits super (a,d)-edge-antimagic total labelings, namely $mK_{1,m} \cup S_{k,1}$ in [2].

More results concerning on super edge antimagic total labeling, see for instances in a nice survey paper by Gallian [6].

Now, we will concentrate on the disjoint union of cycle with cycle with chord, denoted by $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$.

2. Some Useful Lemmas

We start this section by a necessary condition for a graph to be super (*a, d*)-edge- antimagic total, providing a least upper bound for feasible values of *d*.

Lemma 1 *If a (p,q)-graph is super (a,d)-edge-antimagic total then* $d \leq \frac{2p+q-5}{q-1}$.

Proof. Assume that a (p,q)-graph has a super (a,d)-edge-antimagic total labeling $f : V(G) \cup E(G) \to \{1,2,\ldots,p+q\}$. The minimum possible edge-weight in the labeling f is at least $1+2+p+1 = p+4$. Thus, $a \geq p+4$. On the other hand, the maximum possible edge-weight is at most $(p-1) + p + (p+q) = 3p+q-1$. So we obtain $a + (q-1)d \leq 3p+q-1$ which gives the desired upper bound for the difference d. □

The following lemma, proved by Figueroa-Centeno *et al*. in [13], gives a necessary and sufficient condition for a graph to be super edge magic (super $(a,0)$-edge antimagic total).

Lemma 2 *A (p,q)-graph G is super edge-magic if and only if there exists a bijective function $f: V(G) \to \{1,2,\ldots,p\}$ such that the set $S = \{f(u)+f(v): uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a = p+q+s$, where $s = min(S)$ and $S = \{a-(p+1), a-(p+2), \ldots, a-(p+q)\}$.*

In our terminology, the previous lemma states that a (p, q)-graph G is super $(a, 0)$-edge antimagic total if and only if there exists an $(a - p - q, 1)$-edge antimagic vertex labeling. Next, we restate the following lemma that appeared in [8].

Lemma 3 [8] *Let* $\aleph$ *be a sequence* $\aleph = \{c, c+1, c+2, \ldots, c+k\}$*,* k *even. Then there exists a permutation* $\Pi(\aleph)$ *of the elements of* $\aleph$ *such that* $\aleph + \Pi(\aleph) = \left\{2c + \frac{k}{2}, 2c + \frac{k}{2} + 1, 2c + \frac{k}{2} + 2, \ldots, 2c + \frac{3k}{2} - 1, 2c + \frac{3k}{2}\right\}$

3. Disjoint Union of Cycle with Cycle With Chord

We shall write C^k to mean the graph constructed from a cycle C_n by joining two vertices whose distance in the cycle is k [10]. Now, we will study super edge-antimagic of a disjoint union of cycle graph with cycle with chord, denoted by $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$, for $\mu \geq 1, (\mu - 1) \leq m \leq \mu, n \geq 7$ and n odd. It is a disconnected graph with vertex set $V = \{x_i^j | 1 \leq i \leq n, 1 \leq j \leq 2m + 2\mu - 1\}$ and edge set:
$E = \{x_i^j x_{i+1}^j \cup x_n^j x_1^j | 1 \leq i \leq n, 1 \leq j \leq 2m + 2\mu - 1\} \cup \{x_3^j x_{n-2}^j | 1 \leq j \leq m\} \cup \{x_1^j x_{n-2}^j | (m + \mu) \leq j \leq m + 2\mu - 1\}$.
Thus : $p = m(n) + (\mu - 1)(n) + \mu(n) + m(n) = n(2m + 2\mu - 1)$
and $q = m(n+1) + (\mu - 1)(n) + \mu(n+1) + m(n) = n(2m + 2\mu - 1) + m + \mu$.
If the disjoint union of $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$, has a super (a, d)-edge antimagic total labeling then, for $p = n(2m + 2\mu - 1)$ and $q = n(2m + 2\mu - 1) + m + \mu$, it follows from from Lemma 1 that the upper bound of d is $d \leq 3 - \left(\frac{2m+2\mu+2}{2mn+2\mu n-n+m+\mu-1}\right)$, $d \geq 0$, d is integer, so $d \in \{0, 1, 2\}$.

The following theorem describes an $(a, 1)$-edge antimagic vertex labeling for disjoint union of $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$, for $\mu \geq 1$, $(\mu - 1) \leq m \leq \mu$, $n \geq 7$ and n odd.

Lemma 4 *The disjoint union of* $mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$ *has an* $(a, 1)$*-edge antimagic vertex labeling for* $\mu \geq 1$*,* $(\mu - 1) \leq m \leq \mu$*,* $n \geq 7$ *and* n *odd.*

Proof. Define the vertex labeling $\alpha_1 : V(mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n) \rightarrow \{1, 2, \ldots, n(2m + 2\mu - 1)\}$ in the following way:
For x_i^j; $1 \leq i \leq n - 1$, $1 \leq j \leq m + \mu$

$\alpha_1(x_i^j) = \frac{i(2m+2\mu-1)+2j+1}{2} + \left(\frac{(-1)^i+1}{4}\right) n(2m+2\mu-1) + (-1)^i(m+\mu) - \left(\frac{(-1)^i+1}{2}\right)3j + \left(\frac{(-1)^i+1}{2}\right)$

For x_i^j; $1 \le i \le n-1$, $m+\mu+1 \le j \le 2m+2\mu-1$

$\alpha_1(x_i^j) = \frac{(2m+2\mu-1)-2(m+\mu)+2j+1}{2} + \left(\frac{(-1)^i+1}{4}\right) n(2m+2\mu-1) + \left((-1)^i + 1\right)2(m+\mu) - \left(\frac{(-1)^i+1}{2}\right)3j$

For x_n^j; $1 \le j \le m+\mu$

$\alpha_1(x_n^j) = \frac{i(2m+2\mu-1)+2j+1}{2}$

For x_n^j; $m+\mu+1 \le j \le 2m+2\mu-1$

$\alpha_1(x_n^j) = \frac{(2m+2\mu-1)-4(m+\mu)+2j+1}{2}$

The vertex labeling α_1 is a bijective function. The edge weights of $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$, under the labeling α_1, constitute the following sets:

For $x_i^j x_{i+1}^j$; $1 \le i \le n-1, 1 \le j \le 2m+2\mu-1$

$w_{\alpha_1}^1(x_i^j x_{i+1}^j) = \frac{(n+2i)(2m+2\mu-1)+2(m+\mu)-2j+3}{2}$, $1 \le i \le n-2$; $1 \le j \le m+\mu$

$w_{\alpha_1}^2(x_i^j x_{i+1}^j) = \frac{(n+2i)(2m+2\mu-1)+6(m+\mu)-2j+1}{2}$, $1 \le i \le n-2$; $m+\mu+1 \le j \le 2m+2\mu-1$

$w_{\alpha_1}^3(x_i^j x_{i+1}^j) = \frac{3n(2m+2\mu-1)-2j+3}{2}$, $i = n-1$; $1 \le j \le 2m+2\mu-1$

For $x_n^j x_1^j$; $1 \le j \le 2m+2\mu-1$

$w_{\alpha_1}^4(x_n^j x_1^j) = \frac{n(2m+2\mu-1)+4j-1}{2}$, $1 \le j \le m+\mu$

$w_{\alpha_1}^5(x_n^j x_1^j) = \frac{n(2m+2\mu-1)-4(m+\mu)+4j+1}{2}$, $m+\mu+1 \le j \le 2m+2\mu-1$

For $x_3^j x_{n-2}^j$; $1 \le j \le m+\mu$

$w_{\alpha_1}^6(x_3^j x_{n-2}^j) = \frac{n(2m+2\mu-1)-2(m+\mu)+4j+1}{2}$

For $x_1^j x_{n-2}^j$; $m+\mu \le j \le m+2\mu-1$

$w_{\alpha_1}^7(x_1^j x_{n-2}^j) = \frac{n(2m+2\mu-1)-6(m+\mu)+4j+3}{2}$

It is not difficult to see that the set:

$$\bigcup_{t=1}^{7} w_{\alpha_1}^t = \left\{\frac{n(2m+2\mu-1)-2(m+\mu)+3}{2}, \frac{n(2m+2\mu-1)-2(m+\mu)+5}{2}, \dots, \frac{3n(2m+2\mu-1)+1}{2}\right\}$$

consists of consecutive integers. Thus α_1 is a $\left(\frac{n(2m+2\mu-1)-2(m+\mu)+3}{2},1\right)$-edge antimagic vertex labeling. □

Theorem 1 *The disjoint union of* $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ *has a super* $\left(\frac{5n(2m+2\mu-1)+3}{2},0\right)$*-edge antimagic total labeling and a super* $\left(\frac{3n(2m+2\mu-1)-2(m+\mu)+5}{2},2\right)$*-edge antimagic total labeling, for* $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \geq 7$ *and n odd.*

Proof. *Case 1.* for $d = 0$

We have proved that the vertex labeling α_1 is a $\left(\frac{n(2m+2\mu-1)-2(m+\mu)+3}{2},1\right)$-edge antimagic vertex labeling. With respect to Lemma 2, by completing the edge labels $p+1$, $p+2, \ldots, p+q$, we are able to extend labeling α_1 to a super $(a, 0)$-edge antimagic total labeling, where, for $p = n(2m+2\mu-1)$ and $q = n(2m+2\mu-1)+m+\mu$, the value $a = \frac{5n(2m+2\mu-1)+3}{2}$. □

Proof. *Case 2.* for $d = 2$

Label the vertices of $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ with $\alpha_2(x_i^j) = \alpha_1(x_i^j)$ and $\alpha_2(x_n^j) = \alpha_1(x_n^j)$, for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, 2m+2\mu-1$; and label the edges with the following way :

For $x_i^j x_{i+1}^j;\ 1 \leq i \leq n-1, 1 \leq j \leq 2m+2\mu-1$

$\alpha_2 = (n+i)(2m+2\mu-1)+2(m+\mu)-j+1,\quad 1 \leq i \leq n-2\ ;\ 1 \leq j \leq m+\mu$

$\alpha_2 = (n+i)(2m+2\mu-1)+4(m+\mu)-j,\ 1 \leq i \leq n-2\ ;\ m+\mu+1 \leq j \leq 2m+2\mu-1$

$\alpha_2 = 2n(2m+2\mu-1)+(m+\mu)-j+1,\ i = n-1\ ;\ 1 \leq j \leq 2m+2\mu-1$

For $x_n^j x_1^j, 1 \leq j \leq 2m+2\mu-1$

$\alpha_2 = n(2m+2\mu-1)+(m+\mu)+2j-1,\quad 1 \leq j \leq m+\mu$

$\alpha_2 = n(2m+2\mu-1)-(m+\mu)+2j,\quad m+\mu+1 \leq j \leq 2m+2\mu-1$

For $x_3^j x_{n-2}^j, 1 \leq j \leq m$

$\alpha_2 = n(2m+2\mu-1)+2j$

For $x_1^j x_{n-2}^j, m+\mu \leq j \leq m+2\mu-1$

$\alpha_2 = n(2m+2\mu-1)-2(m+\mu)+2j+1$

The total labeling α_2 is a bijective function from $V(mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n) \cup E(mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n)$ onto the set $\{1, 2, 3, \ldots, 2n(2m+2\mu-1)+m+\mu\}$. The edge-weights of $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$, under the labeling α_2, constitute the sets:

For $x_i^j x_{i+1}^j;\ 1 \leq i \leq n-1, 1 \leq j \leq 2m+2\mu-1$

$W_{\alpha_2}^1 = \{w_{\alpha_1}^1 + \alpha_2(x_i^j x_{i+1}^j),\ jika\ 1 \le i \le n-2;\ \ 1 \le j \le m+\mu\}$

$= \left(\frac{n+2i)(2m+2\mu-1)+2(m+\mu)-2j+3}{2}\right) + (n+i)(2m+2\mu-1) + 2(m+\mu) - j + 1$

$= \frac{(3n+4i)(2m+2\mu-1)+6(m+\mu)-4j+5}{2}$

$W_{\alpha_2}^2 = \{w_{\alpha_1}^2 + \alpha_2(x_i^j x_{i+1}^j),\ jika\ 1 \le i \le n-2;\ \ m+\mu+1 \le j \le 2m+2\mu-1\}$

$= \left(\frac{(n+2i)(2m+2\mu-1)+6(m+\mu)-2j+1}{2}\right) + (n+i)(2m+2\mu-1) + 4(m+\mu) - j$

$= \frac{(3n+4i)(2m+2\mu-1)+14(m+\mu)-4j+1}{2}$

$W_{\alpha_2}^3 = \{w_{\alpha_1}^3 + \alpha_2(x_i^j x_{i+1}^j),\ jika\ i = n-2;\ \ 1 \le j \le 2m+2\mu-1\}$

$= \left(\frac{3n(2m+2\mu-1)-2j+3}{2}\right) + 2n(2m+2\mu-1) + (m+\mu) - j + 1$

$= \frac{7n(2m+2\mu-1)+2(m+\mu)-4j+5}{2}$

For $x_n^j x_1^j, 1 \le j \le 2m+2\mu-1$

$W_{\alpha_2}^4 = \{w_{\alpha_1}^4 + \alpha_2(x_n^j x_1^j),\ jika\ \ 1 \le j \le m+\mu\}$

$= \left(\frac{n(2m+2\mu-1)+4j-1}{2}\right) + n(2m+2\mu-1) + (m+\mu) + 2j - 1$

$= \frac{3n(2m+2\mu-1)+2(m+\mu)+8j-3}{2}$

$W_{\alpha_2}^5 = \{w_{\alpha_1}^5 + \alpha_2(x_n^j x_1^j),\ jika\ \ m+\mu+1 \le j \le 2m+2\mu-1\}$

$= \left(\frac{n(2m+2\mu-1)-4(m+\mu)+4j+1}{2}\right) + n(2m+2\mu-1) - (m+\mu) + 2j$

$= \frac{3n(2m+2\mu-1)-6(m+\mu)+8j+1}{2}$

For $x_3^j x_{n-2}^j$, $1 \le j \le m$

$W_{\alpha_2}^6 = \{w_{\alpha_1}^6 + \alpha_2(x_3^j x_{n-2}^j)\}$

$= \left(\frac{n(2m+2\mu-1)-2(m+\mu)+4j+1}{2}\right) + n(2m+2\mu-1) + 2j$

$= \frac{3n(2m+2\mu-1)-2(m+\mu)+8j+1}{2}$

For $x_1^j x_{n-2}^j$, $m+\mu \le j \le m+2\mu-1$

$W_{\alpha_2}^7 = \{w_{\alpha_1}^7 + \alpha_2(x_1^j x_{n-2}^j)\}$

$= \left(\frac{n(2m+2\mu-1)-6(m+\mu)+4j+3}{2}\right) + n(2m+2\mu-1) - 2(m+\mu) + 2j + 1$

$= \frac{3n(2m+2\mu-1)-10(m+\mu)+8j+5}{2}$

It is not difficult to see that the set of :

$\bigcup_{t=1}^{7} W_{\alpha_1}^t = \left\{\frac{3n(2m+2\mu-1)-2(m+\mu)+5}{2}, \frac{n(2m+2\mu-1)-2(m+\mu)+9}{2}, \dots, \frac{7n(2m+2\mu-1+2(m+\mu)+1}{2}\right\},$

contains an arithmetic sequence with the first term $\left(\frac{3n(2m+2\mu-1)-2(m+\mu)+5}{2}\right)$ and common difference 2. Thus α_2 is a super $\left(\frac{3n(2m+2\mu-1)-2(m+\mu)+5}{2},2\right)$-edge antimagic total labeling. This concludes the proof. □

Theorem 2 *The disjoint union of* $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ *has a super* $\left(\frac{4n(2m+2\mu-1)-(m+\mu)+4}{2},1\right)$*-edge antimagic total labeling for* $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \geq 7$ *and n odd.*

Proof. We will prove using Lemma 3. For $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \geq 7$ and n odd, consider the vertex labeling α_1 of the graph $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ from lemma 4 which is a $\left(\frac{n(2m+2\mu-1)-2(m+\mu)+3}{2},1\right)$-EAV labeling. Let sequence $\aleph = \{c, c+1, c+2, \ldots, c+k\}$ be the set of edge-weights of the vertex labeling α_1 for $c = \frac{n(2m+2\mu-1)-2(m+\mu)+3}{2}$ and $k = n(2m+2\mu-1)-(m+\mu)-1$. In light of Lemma 3, there exists a permutation $\prod(\aleph)$ of the elements of $\aleph$ such that $\aleph + \left[\prod(\aleph) + \frac{k}{2} + \frac{m+\mu}{2}\right] = \left\{2c+k+\frac{m+\mu}{2},\ 2c+k++\frac{m+\mu}{2}+1, \ldots,\ 2c+2k+\frac{m+\mu}{2}\right\}$. If $\left[\prod(\aleph) + \frac{k}{2} + \frac{m+\mu}{2}\right]$ is an edge labeling of $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ then $\aleph + \left[\prod(\aleph) + \frac{k}{2} + \frac{m+\mu}{2}\right]$ gives the set of the edge-weights of $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$, which implies that the resulting total labeling is super $\left(\frac{4n(2m+2\mu-1)-(m+\mu)+4}{2},1\right)$-EAT. This concludes the proof. □

4. Conclusion

We have lemma and theorem from bijective function of super (*a, d*)-edge antimagic total labeling on disjoint union of cycle with chord:

- **Lemma 4** *The disjoint union of* $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ *has an* $(a,1)$*-edge antimagic vertex labeling for* $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \overset{n}{\geq} 7$ *and n odd.*
- **Theorem 1** *The disjoint union of* $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ *has a super* $\left(\frac{5n(2m+2\mu-1)+3}{2},0\right)$*-edge antimagic total labeling and a super* $\left(\frac{3n(2m+2\mu-1)-2(m+\mu)+5}{2},2\right)$*-edge antimagic total labeling, for* $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \geq 7$ *and n odd.*
- **Theorem 4** *The disjoint union of* $mC_n^k \cup (\mu-1)C_n \cup \mu C_n^3 \cup mC_n$ *has a super* $\left(\frac{4n(2m+2\mu-1)-(m+\mu)+4}{2},1\right)$*-edge antimagic total labeling for* $\mu \geq 1, (\mu-1) \leq m \leq \mu$, $n \geq 7$ *and n odd*

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